**Point Kinetics Behavior of Molten Salt Reactors Using Delayed Differential Equation Solver in MATLAB**

Connor Bales and Zeyun Wu

*Department of Mechanical and Nuclear Engineering, Virginia Commonwealth University*
*401 W Main St, Richmond, VA 23220*
balescf@vcu.edu; zwu@vcu.edu

## INTRODUCTION

The point reactor kinematic equation (PRKE) is a simplified mathematical model used to describe the fast transient behavior of the neutron and neutron precursor population in a nuclear reactor. The standard PRKE model assumes the neutron distribution is spatially constant and thus the reactor can be simplified to a 'point'. The benefit of this modeling is that it provides the total number of neutrons within a reactor which can then provide information on the thermal power output of said reactor. In the standard PRKE model for conventional reactors like pressurized water reactors (PWR), a set of coupled ordinary differential equations (ODE) are used. In molten salt fueled reactors (MSR), the fuel is dissolved into a working fluid that causes delayed neutron precursors (DNP) to drift out of the core and reenter the core with a delay and at a diminished amount. This delayed neutron loss occurs as the DNP's are carried outside of the core and into a region in which the precursor's decays and neutrons generated in this region cannot effectively contribute to the chain reactions within the system. Additionally, the drifting of the DNP's means that the delayed neutrons and the prompt neutrons will have a different spatial distribution. To count this DNP drift effect in MSR, the conventional ODE based PRKE is modified accordingly and formed as a set of delayed differential equations (DDE) model. This effectively models the total neutron population despite the spatial differences between the prompt neutrons and the delayed neutrons released by the DNP's.

The purpose of this work was to use the built-in MATLAB delayed differential equation solvers to model and predict the point kinetics behavior for nuclear reactors. Specifically, the built-in function, *dde23*, was used to solve the PRKE for a MSR to account for its unique fuel flowing and DNP drifting feature.

## THEORY

The neutron behavior in the ODE based PRKE model is described as

$$\frac{d}{dt}P(t)=\frac{\rho_{net}(t)-\beta}{\Lambda}P(t)+\sum_i \lambda_i C_i(t) \quad (1)$$

where $P(t)$ stands for the power rate, $\rho_{net}$ is the net reactivity of the core, $\beta$ is the effective delayed neutron fraction, $\Lambda$ is the prompt neutron generation time, and $\lambda_i$ is the decay constant for the $i^{th}$ group of DNP, and $C_i$ is the concentration for the $i^{th}$ group of DNP governed by a set of DDE since there is an inlet and outlet for the fuel salt within the MSR core, as shown in Fig. 1.

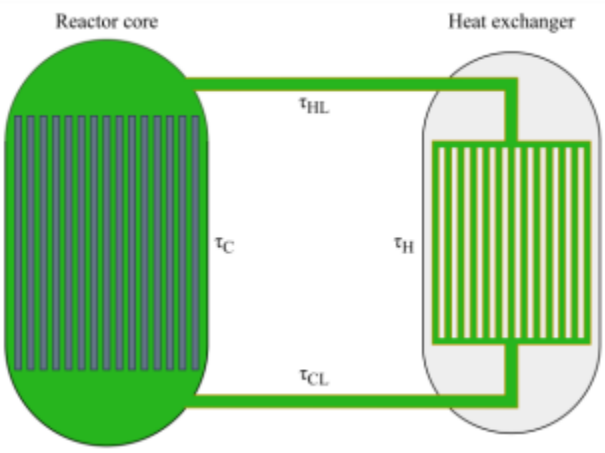


Fig. 1. Schematic of the primary fuel loop in MSR [1].

The DNP concentration equation in the PRKE model is described as the following DDE formulation

$$\frac{d}{dt}C_i(t)=\frac{\beta_i}{\Lambda}P(t)-\left[\lambda_i+\frac{1}{\tau_C}\right]C_i(t)+\frac{e^{-\lambda_i\tau_L}}{\tau_C}C_i(t-\tau_L) \quad (2)$$

where $\beta_i$ is the delayed neutron fraction contributed from the $i^{th}$ group of the DNP, $\tau_C$ is the time taken for the fuel salt to travel through the core, and $\tau_L$ is the time the fuel salt travels outside of the core that is given by:

$$\tau_L=\tau_{HL}+\tau_H+\tau_{CL} \quad (3)$$

where $\tau_{HL}$, $\tau_H$, and $\tau_{CL}$ are the time the fuel salt travels through the hot leg, heat exchanger, and cold leg of the reactor respectively. These time constants are also shown in Fig. 1 and their values are reactor dependent. For the purpose of this experiment, the values of these will match those in the reference solution [2]. As can be seen, Eq. (2) is coupled to

Eq. (1) in that the power dictates the rate of creation of the neutron precursors and vice versa.

The temperatures of the different components within the reactor system are needed to determine the net reactivity of the system because of the thermal feedback. The temperature of the $i^{th}$ node of the fuel salt within the reactor is given by

$$\frac{d}{dt}T_{f,i}(t) = \frac{N}{m_f c_{p,f}}\{\gamma_f f_i P(t) + \frac{k_{f,g}}{N}[T_{g,i}(t) - T_{f,i}(t)] - \dot{m}c_{p,f}[T_{f,i}(t) - T_{f,i-1}(t)]\} \quad (4)$$

where $N$ is the number of nodes the reactor and heat exchanger are split into, $c_{p,f}$ is the heat capacity of the fuel salt, $\gamma_f$ is the fraction of heat generated in the fuel, $f_i$ is the fraction of energy generated in the $i^{th}$ node, $k_{f,g}$ is the fuel to graphite heat transfer coefficient, $T_{g,i}(t)$ is the graphite temperature in the $i^{th}$ node, $T_{f,i}(t)$ is the fuel temperature in the $i^{th}$ node, and $\dot{m}$ is the fuel salt mass flow rate. The graphite moderator temperature is given by

$$\frac{d}{dt}T_{g,i}(t) = \frac{N}{m_g c_{p,g}}\{\gamma_g f_i P(t) - \frac{k_{f,g}}{N}[T_{g,i}(t) - T_{f,i}(t)]\} \quad (5)$$

where $m_g$ is the mass of the graphite, $c_{p,g}$ is the heat capacity of the graphite, and $\gamma_g$ is the fraction of heat generated in the graphite. Lastly the temperature of the fuel salt within the heat exchanger was given by:

$$\frac{d}{dt}T_{h,i}(t) = \frac{N}{m_h c_{p,h}}\{\dot{m}c_{p,h}[T_{h,i}(t) - T_{h,i-1}(t)] - \frac{k_{f,s}}{N}[T_{h,i}(t) - T_{c,i}(t)]\} \quad (6)$$

where $m_h$ is the mass of the fuel salt within the heat exchanger, $c_{p,h}$ is the heat capacity of the fuel salt within the heat exchanger, $k_{f,s}$ is the heat exchanger heat transfer coefficient, $T_{h,i}$ is the temperature of the fuel salt in the $i^{th}$ node, and $T_{c,i}$ is the temperature of the coolant in the $i^{th}$ node, which is assumed to be constant and uniform across nodes. In the case of the zero$^{th}$ nodes of the fuel salt and the heat exchanger, the thermal coupling is defined by

$$T_{f,0}(t) = T_{h,N}(t - \tau_{CL}) \quad (7)$$

and

$$T_{h,0}(t) = T_{f,N}(t - \tau_{HL}) \quad (8)$$

Using Eq.(4) through Eq.(8), the initial temperatures of the fuel salt and the graphite can be obtained by using a steady state condition. Following the calculations for initial temperatures, there is a need to solve for the base reactivity insertion needed due to the outflow of DNP in the MSR [3], which is given by

$$\rho_0 = \frac{\beta(1 - e^{-\lambda^* \tau_L})}{\lambda \tau_C + (1 - e^{-\lambda^* \tau_L})} \quad (9)$$

where $\rho_0$ is the base reactivity and $\lambda$ is average decay constant of the DNP. All these parameters are then integrated into the net reactivity model

$$\rho(t) = \rho_0 + \rho_i(t) + \rho_{fb}(t) \quad (10)$$

where $\rho_i$ is the externally inserted reactivity. In this scenario of nodal thermal couplings [1], the reactivity feedback is given as

$$\rho_{fb}(t) = \sum_{i=1}^{N}(\alpha_f I_{f,i}\Delta T_{f,i} + \alpha_g I_{g,i}\Delta T_{g,i}) \quad (11)$$

where $I_{f,i}$ is the neutron importance factor in the $i^{th}$ node of the fuel salt, and $I_{g,i}$ is the neutron importance factor for the $i^{th}$ node of the graphite section.

## CASE PROBLEM AND RESULTS

In calculating the expected change in power for a given reactivity insertion, the built-in MATLAB solver *dde23* was used to solve the system of delayed differential equations summarized above. To increase the fidelity of the *dde23* function, the solver was set to have a relative tolerance and absolute tolerance of 1E-12. The values of the variables and initial conditions needed for simulating the reactivity insertion accident (RIA) of a typical MSR are respectively summarized in Table A.I and Table A.II shown in the Appendix. The expected power change during the 10 pcm positive reactivity insertion for the given MSR was extracted from a reference paper [2], specifically the "1 Node" solution in Fig.2, while the solutions calculated from the DDE model (i.e., from this work) were compared to the "1 Node" reference solution. Note we only employed a one-node model in the current MATLAB implementation.

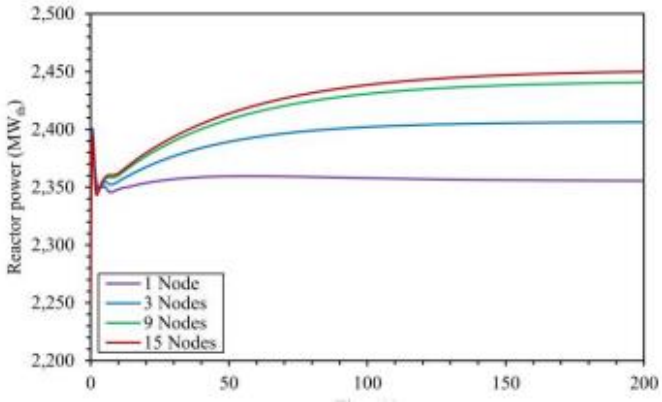


Fig.2. Expected power change with 10 pcm insertion into the given MSR [2].

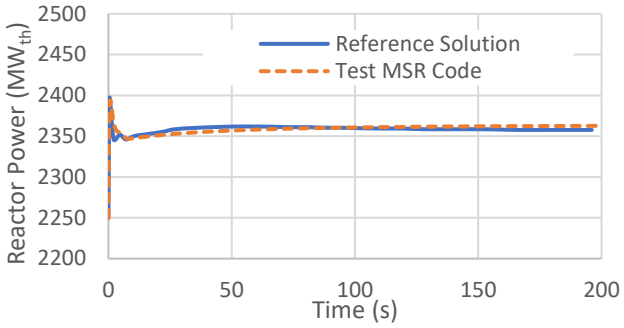


Fig.3. Comparison of power transients over 200 s.

Comparing the power change curves in Fig.3, one can see the general shape and magnitude of the curve between the documented and DDE modeled results are very close, with both having an initial spike followed by a small drop and then a plateau. This, Fig. 3, shows the accuracy of the MATLAB *dde23* function when used to calculate the long-term effects caused by the 10 pcm insertion of reactivity into the reactor. However, there are short term inaccuracies associated with it, as seen in Fig. 4.

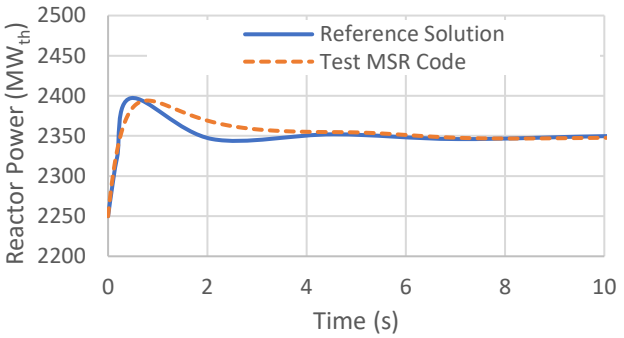


Fig.4. Comparison of power transients over 10 s.

Upon narrowing the time scale, a key difference in when the power spikes peak and the behaviors of the reactor power immediately following the power reaching its peak appears. In comparison to the reference solution, the MATLAB solution peaks slightly after the reference solution and does not dip below its steady state value before reaching steady state. Instead, it gradually declines to its steady state power. This error could originate from how MATLAB handles modeling with a single node in the thermal system or from the chosen delay differential equation tolerance. Despite this, the one-node DDE model of the MSR nearly mirrors the single node model in the Ref. [2].

Despite the inaccuracies in the mentioned topics, the quick generation of a generalized behavior of a reactor upon a reactivity insertion can prove to be valuable for cases where the reactor behavior needs to be predicted within a small window of time and the short-term behavior is negligible.

## CONCLUSION AND FUTURE WORK

This research revealed the benefit of using built-in MATLAB functions as a solver for DDEs in reactor physics as it maintains a moderate degree of short-term accuracy and a high degree of long-term accuracy while having a shorter code run time. The current success lies in how the code replicates the shape and magnitude of the curves, for power and for DNP. The current short comings are that the time at which the value for power peaks is later than the expected results. Further research and analysis into MATLAB *dde23* and PRKE modeling for MSRs need to be conducted to better understand the reasoning for the delay that occurs before the spike in power and into application of a multi-nodal reactors using MATLAB *dde23*.

## APPENDIX

TABLE A.I. Variables used in the given MSR model.

| Symbol | Name | Unit | Value |
|---|---|---|---|
| $\rho(t)$ | Net reactor reactivity | Unitless | Calculated |
| $\rho_i(t)$ | Reactivity inserted | pcm | 10 |
| $\rho_0$ | Critical Reactivity | Unitless | Calculated |
| $\alpha_f$ | fuel feedback coefficient | $\frac{1}{°C}$ | -3.2 x $10^{-5}$ |
| $\alpha_g$ | Moderator (graphite) feedback coefficient | $\frac{1}{°C}$ | 2.35 x $10^{-5}$ |
| $T_{f,i}$ | Fuel temperature at the $i^{th}$ node | $°C$ | Calculated |
| $T_{c,i}$ | Coolant temperature at the $i^{th}$ node | $°C$ | Calculated |
| $T_{g,i}$ | Graphite Temperature at the $i^{th}$ node | $°C$ | Calculated |
| $T_{h,i}$ | HX Salt Temperature at the $i^{th}$ node | $°C$ | Calculated |
| $P(t)$ | Reactor Power | W | Calculated |
| $\beta$ | Delayed neutron fraction | Unitless | Element Dependent |
| $\Lambda$ | Neutron prompt generation time | $\frac{1}{s}$ | $3.6*10^{-4}$ |
| $\lambda$ | decay constant | $\frac{1}{s}$ | Element Dependent |
| $C_i(t)$ | Delayed neutron precursor concentration | atoms | Calculated |
| $\dot{m}$ | Fuel mass flow rate | $\frac{kg}{s}$ | 1.2 x $10^4$ |
| $m_f$ | Mass of fuel | kg | 5.42 x $10^4$ |

| | | | |
|---|---|---|---|
| $c_{p,f}$ | Fuel heat capacity | $\frac{J}{kg*°C}$ | 1357 |
| $m_g$ | Mass of moderator (graphite) | kg | 1.22 x $10^6$ |
| $c_{p,g}$ | Moderator heat capacity (graphite) | $\frac{J}{kg*°C}$ | 1760 |
| $m_h$ | Mass of salt fuel in the HX | kg | 5.38 x $10^3$ |
| $c_{p,h}$ | Fuel heat capacity of fuel salt in the HX | $\frac{J}{kg*°C}$ | 1357 ($c_{p,f} = c_{p,h}$) |
| $k_{f,g}$ | Fuel to graphite heat transfer coefficient | $\frac{W}{°C}$ | 2.48 x $10^7$ |
| $k_{f,s}$ | Heat exchanger heat transfer coefficient | $\frac{W}{°C}$ | 2.32 x $10^7$ |
| $\gamma_f$ | Fraction of heat generated in the fuel | Unitless | 0.97 |
| $\gamma_g$ | Fraction of heat generated in the graphite | Unitless | 0.03 |
| $\tau_C$ | Core transit time constant | s | 4.5 |
| $\tau_L$ | Loop transit time constant | s | 6.0 |
| $\tau_{CL}$ | Cold leg transit time constant | s | 2.1 |
| $\tau_H$ | Heat exchanger transit time constant | s | 1.8 |
| $\tau_{HL}$ | Hot leg transit time constant | s | 2.1 |
| $f_i$ | Fraction of energy generated in the $i^{th}$ node | Unitless | 1 |
| N | Number of nodes | Unitless | 1 |
| $I_{f,i}$ | Neutron importance factor of the fuel salt | Unitless | 1 |
| $I_{g,i}$ | Neutron importance factor of the graphite | Unitless | 1 |

TABLE A.II. Initial conditions given by the steady state.

| Variable | Equation / Value | Unit |
|---|---|---|
| $P(0)$ | 2,250,000 | W |
| $C_1(0)$ | $\frac{\beta_1 * P(0)}{[\lambda_1 + \frac{1}{\tau_C}(1-e^{-\lambda_1 * \tau_L})] * \Lambda}$ | Atoms |
| $C_2(0)$ | $\frac{\beta_2 * P(0)}{[\lambda_2 + \frac{1}{\tau_C}(1-e^{-\lambda_2 * \tau_L})] * \Lambda}$ | Atoms |
| $C_3(0)$ | $\frac{\beta_3 * P(0)}{[\lambda_3 + \frac{1}{\tau_C}(1-e^{-\lambda_3 * \tau_L})] * \Lambda}$ | Atoms |
| $C_4(0)$ | $\frac{\beta_4 * P(0)}{[\lambda_4 + \frac{1}{\tau_C}(1-e^{-\lambda_4 * \tau_L})] * \Lambda}$ | Atoms |
| $C_5(0)$ | $\frac{\beta_5 * P(0)}{[\lambda_5 + \frac{1}{\tau_C}(1-e^{-\lambda_5 * \tau_L})] * \Lambda}$ | Atoms |
| $C_6(0)$ | $\frac{\beta_6 * P(0)}{[\lambda_6 + \frac{1}{\tau_C}(1-e^{-\lambda_6 * \tau_L})] * \Lambda}$ | Atoms |
| $T_f(0)$ | 685.16 | Celsius |
| $T_g(0)$ | 687.88 | Celsius |
| $T_h(0)$ | 546.98 | Celsius |
| $T_c(0)$ | 450 | Celsius |

**REFERENCES**


[1]. T. ABUQUDAIRA, P. TSVETKOV, and P. SABHARWALL, "Exploring the Effect of the Number of Groups of Delayed Neutron Precursors in Reactor Dynamics Simulations of Molten Salt Reactors," *Trans. Am. Nucl. Soc.,* **130**, pp. 1068-1071 (2024).

[2]. T. ABUQUDAIRA, P. TSVETKOV, and P. SABHARWALL, "Neutronics/Thermal-Hydraulics Coupling in Simulation of Advanced Nuclear Reactors," The International Conference on Mathematics and Computational Methods Applied to Nuclear Science and Engineering (M&C 2023), Niagara Falls, Ontario, Canada (2023)

[3]. M. ZAREI, "A refined Time-Delay Modeling of The Molten Salt Reactor Dynamics," *Progress in Nuclear Energy*, **117**, pp. 1–7 (2019).