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A best estimate plus uncertainty safety analysis framework based on RELAP5-3D and RAVEN platform for research reactor transient analyses

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ABSTRACT

This paper develops a best estimate plus uncertainty (BEPU) framework for research reactor transient safety analysis. The BEPU framework is developed based on the system level reactor safety analysis code RELAP5-3D and the data analysis platform RAVEN developed by Idaho National Laboratory. Within the framework, a sensitivity analysis procedure is first conducted to identify the contributions and ranks of individual input parameters to the user-defined figures-of-merit (FOMs) associated with specific transient phenomena. An uncertainty analysis procedure is then performed to quantify the uncertainties of the FOMs resulting from the uncertainties of the input parameters. Many useful outcomes can be realized through the BEPU analysis. Specifically, the sensitivity information obtained from the sensitivity analysis will provide insights about the influence of each different input parameter on FOMs. The uncertainty information obtained from the uncertainty analysis will imply the range of response deviations caused by the propagation of errors existing in various input components.

As a case study for research reactors, the developed BEPU framework was employed to perform design-basis accident (DBA) analysis for one conceptual research reactor design proposed at the National Institute of Standards and Technology (NIST). Two hypothetical DBA scenarios, namely the reactivity insertion accident (RIA) and the loss of flow accident (LOFA), were modeled and analyzed through the BEPU framework. To demonstrate the value of the BEPU framework, the BEPU analysis results were compared to that obtained from the conventional transient safety analysis procedure, which was conducted by using commonly used transient safety analysis codes including RELAP5-3D and PARET. The comparison shows that the BEPU analysis is capable of providing additional sensitivity and uncertainty information that help confirm safety margins of the NIST conceptual research reactor during both RIA and LOFA situations, which justifies the advantages and benefits of the BEPU safety analysis framework developed in this work.

1. Introduction

Nuclear power plants operate contingent upon receiving a favorable safety assessment from their national regulator and safety authority. Among the many facets of nuclear safety is the verification of integral design-basis performance through analysis. Presently, in the commercial industry, both the deterministic and stochastic treatments of uncertainties, which are important to reactor safety, are performed by using best-estimate computer codes like RELAP5-3D (NRC, 2015). In contrast, there are few examples of safety evaluation models applying a stochastic treatment of uncertainty in research reactor community. This is principally because research reactors typically maintain much larger

operational safety margins that provide greater capability for observation. While deterministic methods may be sufficient in the past for research reactor applications, these facilities may be missing out on their full potential by not quantifying key modeling uncertainties and implementing them in their safety evaluation models.

Modeling uncertainties originate from various sources including approximations of computer models, uncertainties in input parameters, limited knowledge to physical phenomena, etc. Due to this reason, according to the US Code of Federal Regulation (CFR) Title 10, Part and Section 50.46 (NRC, 2012) and related regulatory guidance documents, nuclear safety analysis evaluation models are mandated to identify and analytically address the uncertainties. In compliance with these practice

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trends and standards, the stochastic approach based best estimate plus uncertainty (BEPU) methodology has been found acceptable by nuclear regulatory bodies for commercial applications based on the completeness provided by this enhanced knowledge and understanding of biases and uncertainties (Martin et al., 2019; Cacuci, 2019).

Applying the stochastic approach based BEPU methodology to reactor safety analysis generally consists of the following three parts: physics model development, sensitivity and uncertainty analysis module establishment, and accident scenario abstraction and analyses. Among them, the stochastic features are mainly affiliated with the sensitivity and uncertainty analysis, in which one needs to determine the figures-ofmerit (FOMs) from outputs as well as specify the uncertainties from input parameters in terms of probability distribution functions (PDF). Random sampling approach is generally applied to produce perturbed input samples, and to determine the uncertainties of the outputs to a certain level of tolerance limits. In the stochastic sampling approach, one sample is a combination of various input parameters and known as one realization of the problem. At the preparation stage of the BEPU method, the phenomenon identification and ranking table (PIRT) is usually produced in order to systematically identify phenomena that are of both high importance and high uncertainty, and thus of primary interest for further studies (NEA, 2018; Glaeser, 2008; Martin, 2011). The PIRT that is appropriate to the studies of interest is normally established through thoughtful discussions of a panel of experts in the area. Since the focus of this paper is to demonstrate the feasibility of the proposed BEPU framework by analyzing two typical design-basis transients in a research reactor, we did not use the PIRT process to identify and rank the importance of phenomena in the current work, but rather determined the importance based on the understanding of the fundamental physics of the modeled phenomena, and performed a simple sensitivity analysis to rank the input parameters of interest.

The role of the uncertainty analysis in BEPU evaluation models is to characterize the uncertainty in a system response due to the uncertainty in the input parameters of the physics model, while the sensitivity analysis aims to determine the contributions of each individual input parameter to a specific system response (Helton et al., 2006). The first step in the course of uncertainty analysis is to identify and characterize the source of uncertainty. The International Atomic Energy Association (IAEA) has summarized the source of uncertainty in a typical reactor safety analysis into five categories (IAEA, 2000): code or model uncertainty, representation uncertainty, scaling uncertainty, plant uncertainty, and user-caused uncertainty. The characterization of these sources of uncertainty begins by identifying the elements that fall into the categories of either epistemic or aleatoric uncertainty (Helton et al., 2006). The epistemic uncertainty is attributed to the lack of knowledge of the appropriate value to use, whereas the aleatoric uncertainty is purely caused by the inherent statistical nature of the physical system. In this work, we simply focus on the aleatoric uncertainties because the epistemic uncertainties are relatively more difficult to quantify and normally require more in-depth and broader knowledge to improve the physics model. Specifically, the input uncertainties existing in boundary and initial conditions of physics models can be considered as plant uncertainties, and user-caused uncertainties are covered by input uncertainties due to their random natures. The uncertainties of the input parameters can be characterized by specific probability distribution functions within their uncertainty ranges defined. Sampling-based Monte Carlo (MC) approach can be employed to perform the uncertainty analysis in this work because the stochastic characteristics of the MC approach are quite suitable for the BEPU methodology (Helton et al., 2006).

The BEPU methodology has been widely used for reactor safety analysis and nuclear power plant licensing. The first well known BEPU framework was developed through the Code Scaling, Applicability, and Uncertainty (CSAU) methodology in the late 1980s (Boyack, 1989). Since then, several BEPU realizations based on CSAU have been established for reactor safety analysis, including ASTRUM (Frepoli et al.,

2004), AREVA's model (Martin and O'Dell, 2005) and TRACG (Sarikava et al., 2008). More recently, the BEPU philosophy has been adopted by Marcum and Brigantic (2015) with a comprehensive uncertainty and sensitivity analysis for the Multi-Application Small Light Water Reactor using RELAP5-3D and VIPRE-01 (Cuta et al., 1985). The quantitative comparisons of the results from these two codes showed an equivalency for both codes. Besides the reactor safety analysis, the BEPU methodology has also been applied for fuel performance analysis. Ikonen (2016) performed a comparison of several global sensitivity analysis methods under the BEPU framework in his LWR fuel performance analysis using FRAPCON code (Geelhood et al., 2011). Similarly, Brown and Zhang (2016) performed an uncertainty quantification and sensitivity analysis with the CASL core simulator VERA-CS (Virtual Environment for Reactor Applications - Core Simulator). In this work, the BEPU approach was exercised by establishing a fuel assembly model and performing uncertainty quantification by considering fourteen input parameters. Minimum departure from nucleate boiling ratio (MDNBR), maximum fuel center-line temperature, and maximum outer clad surface temperature were chosen as the FOMs in the analyses. Pearson and several other correlation coefficients were calculated and used in the sensitivity analysis. Through these applications, the BEPU method was proven to be a reliable tool to help identify the most influential parameters and quantify uncertainties associated with the sensitive parameters.

Though the BEPU approach is more commonly applied in safety analysis for large size water reactors, it has not been wildly used for research reactor analysis. In this paper, we made a similar effort as some work described above and developed a BEPU framework with the specific goal for the transient safety analysis of research reactors. While research reactors are generally regarded as being more safe in terms of thermal-hydraulics (T/H) safety margins, there are many unique safety concerns for research reactor such as more frequent operation and refueling and higher neutron flux level for those high performance research reactors. The BEPU method represents the state-of-the-art computational modeling technique in reactor transient safety analysis, thus it brings up many interests in research reactor community to understand the potentials and influences of this method to research reactor safety analysis.

The BEPU framework developed in this paper was built upon RELAP5-3D and the Risk Analysis Virtual Environment (RAVEN) (Alfonsi et al., 2016) developed by Idaho National Laboratory (INL). Under the BEPU framework, a sensitivity analysis procedure was first conducted to identify the contributions and ranks of input parameters to the uncertainty of the user-defined FOMs. The uncertainty analysis procedure then followed to quantify the uncertainties of a specific figure-of-merit (FOM) caused by the uncertainties in the input parameters. The sensitivity analysis aims to provide insights of the levels of influence from different input parameters, and the uncertainty analysis aims to assess the response deviations caused by input parameter uncertainties. Both information will enhance the physics understanding of the reactor transient procedure under investigation.

For demonstration, the developed BEPU framework was employed to perform design-basis accident (DBA) analysis for one conceptual research reactor design proposed at the National Institute of Standards and Technology (NIST). Two hypothetical DBA scenarios, namely the reactivity-insertion accident (RIA) and loss of flow accident (LOFA), were modeled and analyzed through the BEPU framework. To demonstrate the advantages of the BEPU framework, the BEPU analysis results were compared to that obtained from the conventional deterministic transient safety analysis evaluation model. This analysis was conducted by using commonly used transient safety analysis codes including RELAP5-3D and PARET (Woodruff and Smith, 2001) developed by Argonne National Laboratory. The BEPU analysis results are expected to provide additional information that quantifies uncertainties of the code predictions as well as warrant high level confidence to the safety margins of the conceptual NIST reactor during both RIA and LOFA scenarios, which further verifies the advantages and benefits of the BEPU frame

developed in this work.

The rest of the paper is organized as follows. Section 2 outlines the methodology and components involved in the BEPU safety analysis framework development. The case study research reactor that will be investigated in the BEPU framework, namely the NIST conceptual designed reactor, is also described in this section. Section 3 and Section 4 discuss the procedure and results of the traditional analysis and BEPU analysis of the case study problem, respectively. Both steady-state condition and two design-basis accidents are investigated in the studies. Section 5 offers some concluding remarks and future perspectives of the current work.

2. BEPU safety analysis framework

This section outlines the methodology of the BEPU safety analysis framework, which basically consists of a physics prediction model (e.g., the system level safety analysis model) and data analysis components (e. g., the sensitivity and uncertainty analysis modules). In this work, the physics prediction capability was realized by using both the multichannel T/H safety analysis code PARET (Woodruff and Smith, 2001) and the system level code RELAP5-3D (NRC, 2015). The sensitivity and uncertainty analysis procedures were accomplished by the RAVEN platform (Alfonsi et al., 2016), which integrates the physics model and manages the uncertainty propagation through the computational model. FOMs and input parameters of interest for the sensitivity and uncertainty analysis were identified at the beginning of the BEPU analysis. For illustration, a case study on safety analysis of the NIST conceptual research reactor design is served as a paradigm through the BEPU framework development and demonstration.

2.1. Description of the case study reactor

The conceptually designed NIST research reactor (Wu et al., 2017) was chosen as a case study reactor to demonstrate the analysis capability of the BEPU framework. The new NIST reactor design considers 20 MW thermal power and a 30-day operating cycle. It is designed as a beam-type research reactor with the primary purpose of delivering high quality neutron sources. The material test reactor (MTR) type plate fuel with low-enriched uranium (LEU) - uranium silicide - was used in the design. Note the uranium silicide fuel is not the designated candidate LEU fuel to convert the existing NIST reactor (e.g., the NBSR). Currently, NIST plans to use the U–10Mo monolithic fuel (Diamond et al., 2015) and has already submitted a preliminary safety analysis report (PSAR) to the U.S. Nuclear Regulatory Commission (NRC) for the fuel conversion for the NBSR (NIST, 2010).

Fig. 1 gives a three-dimensional (3-D) cutaway view of the main components of the conceptual NIST reactor design (Liu and Wu, 2019). A typical tank-in-pool type design pattern was adopted for the design. The reactor core is cooled by a forced downward circulation of light water and surrounded by heavy water in a cylindrical tank. The heavy water reflector tank is about 2.5 m in diameter and 2.5 m in height and placed in the center of a larger light water pool that serves as thermal and biological shields. One unique feature of the design is the reactor core is configured with horizontally split two halves to render strong flux traps to accommodate high-performance cold neutron source (CNS) (Wu et al., 2015). A more detailed description of the core and reactor configuration of the conceptual NIST reactor design can be found in Ref. (Wu et al., 2017), and thus will not be repeated here. The parameters needed for the reactor safety analysis of the reactor are summarized in Table 1, which includes fuel element geometry data, thermal-hydraulics properties and boundary conditions, and reactor kinetics parameters (Liu and Wu, 2019).

2.2. Physics prediction models

Two safety analysis codes were employed to develop physics



Fig. 1. A 3-D cutaway view of the conceptual NIST research reactor (Liu and Wu, 2019).

Table 1

Parameters of the case study reactor (Liu and Wu, 2019).

Fuel element geometry	
Number of fuel elements	18
Number of fuel plates per element	17
Fuel plate width (cm)	6.665
Fuel meat width (cm)	6.134
Fuel plate thickness (cm)	0.127
Fuel meat thickness (cm)	0.066
Cladding thickness (cm)	0.0305
Fuel plate length (cm)	60
Fuel meat length (cm)	67.28
Thermal-hydraulics properties	
Fuel thermal conductivity (W/m·K)	48
Cladding thermal conductivity (W/m·K)	180
Fuel volumetric heat capacity (J/m ³ ·K)	2.225E+6
Cladding volumetric heat capacity (J/m ³ ·K)	2.419E+6
Inlet coolant temperature (°C)	37
Core outlet pressure (kPa)	200
Total thermal power (MW)	20
Inlet volumetric flow rate (gpm)	8000
Hydraulic diameter (cm)	0.56
Hot channel peaking power factor	1.99
Reactor kinetics parameters	
Prompt neutron generation time (µs)	252.63
Effective delayed neutron fraction (β_{eff})	0.00718

prediction models in the BEPU framework, namely the PARET and RELAP5-3D. In our previous work, design-basis accident analyses have been conducted on the case study reactor to assess the T/H performance using PARET (Wu et al., 2016). The ANL PARET code is a computational T/H safety analysis tool with particular suitability for plate-type research reactor (Woodruff and Smith, 2001). However, PARET is merely a channel analysis code and unable to model complete cooling loops in the reactor. The more sophisticated system code RELAP5-3D was considered in this work to improve the model prediction capabilities. In addition, using two independent codes to provide physics predictions in the BEPU framework is envisioned to benefit us in three folds.

First, the RELAP5-3D simulations with nominal input parameters provide reference solutions to the hypothetical accident transients under investigation. Second, the RELAP5-3D model can function as a forward model to enable sensitivity and uncertainty analysis. Last, the anticipated existent discrepancies between the predictions of RELAP5-3D and PARET stand as one typical *epistemic* uncertainty source due to the model differences. The uncertainty assessment procedure described in Section 2.3 will attempt to quantify these uncertainties and provide the best estimates through the BEPU framework.

Fig. 2 shows the nodalization of the RELAP5-3D model (Liu and Wu, 2019) for the case study (e.g., the NIST conceptual reactor design). A multi-channel PARET model based on this nodalization was also established (Wu et al., 2016). The model only focuses on the reactor core region at this moment. As shown in Fig. 2, the hydrodynamics component of the reactor core consists of one hot channel (No.100), one average channel (No.110), and one bypass channel (No.120). The hot channel represents the flow channel with the hottest power peaking factor in the fuel assembly; , and the remaining channels are lumped to one average channel. The bypass channel is developed to consider the flow in the region between fuel assemblies. The upper plenum (No.130) and bottom plenum (No. 160) are modeled to connect and mix the flow at the entrance and exit point of the flow channels. The inlet plenum (flow source) is modeled with a time-dependent control volume (No.140) and the corresponding time-dependent junction. Similarly, the outlet plenum (flow sink and system pressure) is defined by a single control volume (No.180) and the corresponding single junction. No.170 represents the reactor water pool and one natural circulation valve (NCV) is modeled by the TRPVLV component in RELAP5-3D. The NCV will trip in the case of LOFA to enable natural circulations. Since the primary coolant loop has not yet been fully modeled, the core channel is bounded with inlet and outlet components with proper boundary conditions. The heat structure components were included to accommodate the proper heat power profiles of the core, and the power peaking factor for the hot channel was 1.99, which was obtained from the neutronics



Fig. 2. Nodalization of the reactor core in the RELAP5-3D model (Liu and Wu, 2019).

calculations of the reactor model (Wu et al., 2017).

2.3. Sensitivity and uncertainty analysis modules

The sensitivity and uncertainty analysis are essential in the BEPU framework because the best estimate calculation with quantified uncertainties is literally the ultimate goal that drives the reactor safety research and development programs (D'Auria et al., 2008), which also essentially motivates the development of the BEPU safety analysis framework. In this work, the sensitivity and uncertainly analysis are realized by taking advantage of the powerful data analysis capabilities built in the RAVEN platform. The INL RAVEN code is a flexible and multi-purpose data analysis toolset that can be used for sensitivity analysis, uncertainty quantification, regression analysis, probabilistic risk assessment, model optimization, and so on. Depending on the tasks to be accomplished and on the probabilistic characterization of the problem, RAVEN is capable of capturing the uncertainties of the response of the system under consideration by stochastically sampling its own parameters. In terms of the reactor safety analysis, one intriguing feature about RAVEN is that many system modeling software including RELAP5-3D are coupled to RAVEN by either direct (software coupling) or indirect (message coupling) integration approaches. Moreover, the uncertainty data generated by the sampling process can be automatically analyzed by RAVEN using classical statistical and more advanced data mining approaches. Thanks to all these salient features, RAVEN is considered as an ideal tool to be used to assess the sensitivities and quantify the uncertainties for the research reactor transients, and thus heavily used in the BEPU framework development. A more detailed description on coupling the physics models to the RAVEN data analysis modules will be provided in the following sub-sections.

2.4. Figures of merit and input parameters

The FOMs are known to be of critical importance in the scenario analyses, and thus must be identified appropriately in order to enable the sensitivity and uncertainty analysis in the BEPU framework. Considering the focus of this study is for research reactor design-basis accident analysis, particularly for the protected RIA and LOFA, the FOMs selected in this study are peak cladding temperature (PCT) and peak coolant temperature (PCoT). Here the term 'peak' is in the sense of the location in the reactor core where the maximum temperature occurred. According to the acceptance criteria described in US 10 CFR 50.46 (NRC, 2012), both of the two FOMs are known to be of critical importance in evaluating the reactor core safety in the accidental scenarios.

The input parameters that are expected to mostly influence the FOMs need to be identified before the uncertainty analysis. As mentioned earlier, the PIRT methodology (NEA, 2018; Glaeser, 2008; Martin, 2011) are widely used to identify and limit the input parameters in a quantitative manner in the traditional CSAU based BEPU framework (Boyack, 1989). Considering the primary purpose of this study is to demonstrate the proposed BEPU framework for research reactor transient analysis and there is no operation data nor reference PIRT available for the conceptual NIST reactor under investigation, the input parameters considered in this work are not determined through PIRT, but rather chosen based on the understanding of physics models discussed in Section 2.2. The input parameters used in the case study include the initial inlet coolant temperature, the inlet coolant mass flow rate, and the reactor core power. A normal distributed uncertainty was assumed for each of these input parameters and the corresponding uncertainty ranges were defined based on a basic engineering judgement, in which the temperature and flow rate were assumed to have $\pm 10\%$ uncertainty while the power has $\pm 2\%$ uncertainty. The nominal values of the selected input parameters and the distribution with possible perturbation range of uncertainties associated with the input parameters are summarized in Table 2.

Table 2

Input parameter and uncertainty range.

Input parameters	Nominal value	Uncertainty range	Distribution
Inlet coolant Temperature [°C]	37.00	$\pm 10\%$	Normal
Inlet coolant mass flow rate [kg/s]	516.83	$\pm 10\%$	Normal
Reactor core power [MW]	20.00	$\pm 2\%$	Normal

2.5. Overview of the BEPU framework

In the current development, the BEPU analysis capability is achieved by connecting the RELAP5-3D to sensitivity and uncertainty analysis modules through input and output message exchanges. The whole framework is built upon the RAVEN platform, in which RAVEN works in a role of a simulation controller of the RELAP5-3D model by using monitored variables and controlled parameters, while RELAP5-3D acts in the role of a working engine accomplishing physics simulation and delivering predictions. Fig. 3 illustrates the interplay mechanism between RAVEN and RELAP5-3D in the BEPU framework.

The uncertainty analysis within the BEPU framework is carried out as follows. The input parameters and their corresponding uncertainties were compiled in an XML (Extensible Markup Language) file, which works as an active interface to exchange the input and output messages between RELAP5-3D and data processing modules embedded in RAVEN. The input parameters can be automatically perturbed by RAVEN based on their uncertainty information, and random samples of each parameter are generated at the user's command. In the following step, RAVEN engines generate multiple RELAP5-3D input files, execute each input file separately, and deliver the output files in sequence. These outputs can then be used to analyze the uncertainty analysis procedure, MATLAB based data processing utilities are developed to process the I/O streaming data, as indicated in Fig. 3.

For the case study of the NIST reactor, two different transient analysis codes, namely RELAP5-3D and PARET, were employed to demonstrate possible prediction differences (e.g., uncertainties). These differences will then be well interpreted under the BEPU framework. Two design-basis accidents, namely the protected RIA and protected LOFA, are modeled and analyzed as two representative transient accidents to justify the application of the BEPU framework. The detailed description for RIA and LOFA will be provided in Section 3. The FOMs and input parameters used for the case study has been addressed in Section 2.4. With the identified FOMs and input parameters, the random sampling approach and parameter perturbation method discussed above can be employed to investigate the uncertainty structure of the model under investigated.

As a concluding remark to this section, the developed BEPU analysis framework shown in Fig. 3 is capable of examining the uncertainties of the input parameters on the simulation results of the conventional thermal-hydraulic analysis code (such as RELAP5-3D, PARET, etc.) and offering the best estimate of the reactor safety performance characteristics.

3. Conventional transient analysis on the case study

Traditional deterministic method based safety transient analyses for the case study reactor were performed in this section to provide comparative study results to justify the benefits of the BEPU method. As described early, transient analyses were prepared for two hypothetical design-basis accidents (RIA and LOFA) with two conventional physics codes (RELAP-3D and PARET). The analysis starts with a brief description of each accidental situation, followed with a presentation of fruitful results obtained through the conventional analysis procedure. The discussion emphasizes the prediction discrepancies of the two physics codes employed in the analysis, with the motivation of identifying the flaws in the traditional analysis approach and highlighting the needs of BEPU analysis method that will be discussed in Section 4. Some results presented in this section may have appeared in our previous NURETH conference paper (Liu and Wu, 2019). They are repeated here for easy illustration and research completeness. Because both transient accidents of interest were presumably initiated at a steady-state reactor operation condition, some key T/H performance characteristics at steady-state are assessed first in this section.

3.1. Steady-state conditions

The reactor is assumed to operate at the power of 20 MW (full power) in the steady-state operation conditions. The steady-state results were respectively obtained from RELAP5-3D and PARET, and compared here to cross verify the correctness of the modeling procedure and outcomes



Fig. 3. The data flow chart in the BEPU framework based on the RAVEN platform.

of both codes. Fig. 4 shows the axial distributions of cladding surface temperature and coolant temperature in the hot channel and average channel of the end-of-fuel-cycle core, while Table 3 provides a more quantitative comparison of the peak temperature predictions from both codes. These results indicate the temperature of cladding and coolant predicted by both codes agree very well.

3.2. Reactivity insertion accident

The RIA assumes a rapid large positive reactivity (1.5 \$) inserted into an initially critical core operated at the steady state to mimic the control rod ejection accident. During the RIA, the reactor SCRAM occurs when the reactor operates at an overpower of 24 MW (120% of full power). The control rods are assumed to be inserted with a speed of 1.2 m/s for reactor trip with a time delay of 25 ms, which accounts for the reaction time needed by mechanical and electronic circuit operations. Note all reactivity feedback effects are assumed negligible and period trips are not considered in the study. The core state is considered at the end of fuel cycle, which is of particular interest to reactor safety analysis because control rods are all out at this state.

Fig. 5 shows the power transient behavior in the RIA simulated by both codes. The large positive reactivity is assumed to be inserted at 1 s into the operation. As shown in Fig. 5, the reactor reaches a maximum power of \sim 26 MW in factional seconds after the insertion, and then rapidly decreases to the decay heat power level due to the reactor SCRAM tripped when the power exceeds 24 MW. Fig. 6 shows the PCT and PCoT changes in the RIA, respectively. At the transient analysis stage, only the temperatures in the hot channel were closely examined. Table 4 presents a quantitative comparison of the maximum power, peak clad temperature, peak coolant temperature, and their corresponding occurrence time in the RIA. All these results indicate that the predictions of RELAP5-3D and PARET agree well in this scenario.

3.3. Loss of flow accident

Since the model for the primary coolant loop is not available, the LOFA assumes the flow reduction due to the pump coastdown follows an exponential function $\exp(-t/\tau)$, where τ is the time constant of the flow rate decay. In this study, the time constant τ is set to be 1 s to mimic a fast flow loss scenario. The reactor is assumed to operate initially at steady-state conditions. During the LOFA, the reactor SCRAM is tripped by a low coolant flow signal when the coolant flow reaches 85% of its normal

operation value. All other conditions of LOFA are configured the same as RIA.

Fig. 7 shows the transient behaviors of the power and mass flow rate of the hot channel during the LOFA. The results generated by the PARET and RELAP5-3D models are presented in a comparable manner. The LOFA takes place at the initial time (t = 0). The reactor trip occurs at \sim 0.5 s in both codes when the flow rate decreases to 85% of its normal value. As shown in Fig. 7(a), the power rate reduces to the decay heat level nearly immediately after the reactor shutdown. In the RELAP5-3D model, the NCV opens at 2.5 s when flow reaches 10% of its normal flow, then the buoyancy force starts to drive the natural circulation of the flow between the core and the pool. As shown in Fig. 7(b), the reversal flow is quickly established and reaches a stable level of \sim 0.047 kg/s in the hot channel after \sim 5 s into LOFA. Compared to the RELAP-3D results, the flow reversal was predicted slightly earlier in the PARET code.

Fig. 8 shows the changes of peak cladding and peak coolant temperatures in the LOFA predicted by both codes. As shown in Fig. 8, the temperatures of the cladding and the coolant initially increase steeply because of the rapid LOFA at the very early period of the accident. The temperatures reach their first peak values in fractional sections. After the reactor trip at ~0.5 s, both temperatures start to decrease sharply and arrive at minimum values shortly after 1–2 s into the accident. Then the temperatures start to increase again due to the reduction of heat removal and the accumulation of decay heat in the core. The second peak values are observed for both the cladding and the coolant temperatures after the flow reversal occurs and natural circulation is established. In both codes, the temperatures are shown to be decreasing mildly after the establishment of stable natural circulation.

Table 5 provides a quantitative summary of the first and second peak values for PCT, PCoT and their corresponding occurrence time in the LOFA. As shown in Table 5, the maximum cladding temperature and coolant temperature during the entire transient predicted by RELAP5-3D are 123.81 °C and 108.77 °C, respectively, and by PARET are 128.67 °C and 106.76 °C, respectively.

Though the LOFA transients predicted by RELAP-3D and PARET exhibit acceptable agreements in most of transient period, the temperature transients from the two codes have shown some appreciable differences, particularly for the cladding temperatures in the period before and after the stabilized natural circulation. As shown in Fig. 8, the temperatures in PARET decrease sharply after the flow reversal, while RELAP5-3D predicts the temperatures decrease smoothly. These differences may be attributed to the differences in the natural circulation



Fig. 4. Axial temperature distributions of the hot (a) and average channel (b) at the steady-state condition.

Table 3

Peak temperatures in the steady-state condition.

Properties	berties Hot Channel		Average Channel			
	RELAP5-3D	PARET	Deviation	RELAP5-3D	PARET	Deviation
PCT [°C] PCoT [°C]	89.60 53.74	90.10 53.66	0.55% 0.15%	68.96 46.54	69.43 46.50	0.68% 0.09%



Fig. 5. Power transient behavior in the RIA.

modeling mechanism of both codes. In particular, RELAP5-3D takes the momentum change effect owing to the opening of NCV into account, which cannot be modeled in PARET because of the lack of the loop modeling capability.

3.4. Remarks on the conventional transient analysis

The results of the conventional transient analysis indicate the

RELAP5-3D predictions can reach an overall good agreement with the PARET predictions, which verifies the correctness of both models. However, the uncertainties of the system parameters and differences of computational models would play a role in some time periods of the transients, which leads to some prediction discrepancies that are very hard to interpret without a more in-depth understanding of the physics models and broader knowledge to the transient phenomena. The incompleteness of the physics model and lack of knowledge of physics phenomena basically drive the need for the BEPU analysis, which essentially intends to account these defects by integrating the physics predictions with aleatoric uncertainties.

4. BEPU analysis on the case study

The BEPU analysis on the case problem is therefore proceeded with the objective to provide further insights into the transient analysis and quantify the uncertainties associated with the transients. Continuing with conventional safety analysis and using the modules developed under the BEPU framework, the BEPU analysis starts with a sensitivity analysis procedure aiming to determine the contributions of individual input parameters to the uncertainty of designated FOMs and provide insights on the level of influence for each parameter. An uncertainty analysis procedure then followed to determine the uncertainty of FOMs caused by the uncertainty of input parameters. Both the sensitivity and uncertainty analysis offer complementary information to the

Table 4

Maximum values and corresponding occurring times in the RIA.

Properties	RELAP5-3D	PARET	Deviation
Peak power [MW]	26.47	26.51	0.07%
Peak power time [s]	1.13	1.14	0.88%
Reactor trip time [s]	0.01	0.01	0.00%
PCT [°C]	99.27	102.58	5.17%
PCT time [s]	1.18	1.18	1.67%
PCoT [°C]	57.15	57.44	0.50%
PCoT time [s]	1.18	1.20	1.67%



Fig. 6. Variations of peak cladding (a) and peak coolant (b) temperature in the RIA.



Fig. 7. Transient behaviors of power (a) and mass flow rate (b) in the LOFA.



Fig. 8. Variations of peak cladding (a) and peak coolant (b) temperature in the LOFA.

Table 5 Maximum peak temperatures and corresponding occurring times in the LOFA.

Properties	RELAP5-3D	PARET	Deviation
1st PCT [°C]	100.25	104.57	4.13%
1st PCT time [s]	0.50	0.40	25.00%
1st PCoT [°C]	59.47	59.72	0.42%
1st PCoT time [s]	0.50	0.40	25.00%
2nd PCT [°C]	123.81	128.67	3.78%
2nd PCT time [s]	7.50	8.00	6.25%
2nd PCoT [°C]	108.77	106.76	1.88%
2nd PCoT time [s]	8.00	8.00	0.00%

conventional approaches and promote the reactor safety study to a level of higher confidence. Some preliminary results of the current study have been presented in the recent international conference of nuclear engineering (ICONE) meeting (Liu and Wu, 2020), but the work presented in this paper has been significantly expanded since the ICONE meeting. In particular, the uncertainty analysis results for the RIA case are all new and have not been published before.

4.1. Sensitivity analysis

Under the BEPU analysis framework described in Section 2, the sensitivity calculations may be readily accomplished using the sensitivity analysis tools embedded in the RAVEN platform. However, considering the small number of input parameters and the selected FOM response variables for the case study, the relative sensitivity (i.e., the sensitivity coefficient) of the FOM with respect to the input parameter were calculated in a straightforward manner using the following finite difference scheme

$$\alpha = \frac{x_0}{R_0} \frac{\partial R}{\partial x} \approx \frac{x_0}{R_0} \frac{R(x+h) - R(x-h)}{2h}$$
(1)

where α is the relative sensitivity, *R* stands for a generic FOM response variable, *x* represents one specific input parameter, and *h* is the perturbation size to the input. The symbols with subscript 0 indicate the nominal values of the quantity.

Since Eq. (1) is a numerical approximation of the sensitivity coefficient, truncation errors are inevitably introduced with this scheme. To understand the accuracy of the sensitivity estimations by Eq. (1), the sensitivities were evaluated with various perturbation sizes. Fig. 9



Fig. 9. Sensitivity coefficients of PCT (a) and ranks of PCT (b).

depicts the sensitivity information of the steady-state PCT with respect to the three input parameters listed in Table 2 with various perturbations. The perturbations were made randomly in a range of -10% to +10% of the nominal values. As indicated in Fig. 9(a), the sensitivities of PCT to the mass flow rate and reactor power exhibit nearly linear variation characteristics, while the sensitivity of PCT to the inlet temperature shows some non-linearity, which implies that higher-order sensitivity coefficients may be required to fully capture its characteristics. Fig. 9(b) ranks the sensitivities of PCT to the selected three input parameters using the sensitivity values obtained with a very small perturbation. The dimensionless sensitivity coefficients of PCT to inlet coolant temperature, reactor power, and inlet coolant mass flow rate are 0.572, 0.133, and -0.112, respectively. The positive sensitivity simply implies the increase in the value of the parameter will cause the FOM increase and a negative sensitivity means the change will be opposite. These results reveal the inlet coolant temperature is the dominant factor that affects the PCT, compared to the other two parameters.

In a similar manner, Fig. 10 (a) shows the sensitivity coefficients of the steady-state PCoT to the three input parameters with a range of perturbations. Fig. 10(b) shows the rank of the sensitivity coefficients. The sensitivity coefficients of PCoT to inlet coolant temperature, reactor power, and inlet coolant mass flow rate are 0.857, 0.03 and -0.03, respectively, indicating again the inlet coolant temperature is the

dominant factor that affects PCoT among the three input parameters.

4.2. Uncertainty analysis

The sampling-based uncertainty analysis is generally applied in the BEPU framework, so is the uncertainty analysis module in RAVEN employed in this work. The minimum number of sampled calculations needed to meet the two-sided tolerance limit in uncertainty analysis can be determined by using Wilks' formula (Wilks, 1941)

$$\beta = 1 - \gamma^n - n(1 - \gamma)\gamma^{n-1} \tag{2}$$

where β is the confidence level, γ is the probability, and *n* is the minimum number of sampled calculations. Based on the Wilk's formula, to have the uncertainty 95% probability within 95% confidence level (e.g., 95/95 level), the minimum number of sampled calculations is 93. For the 99/99 level, the number is 662. This indicates that Wilks' criteria only requires a relatively small sample size to fulfill the statistics satisfactory. Considering the computational cost of our sample calculation for either RIA or LOFA is nearly negligible (a few seconds in clock time) and RAVEN is efficient in managing parallel computations, we performed 1000 calculations for all cases in the uncertainty analysis to avoid unexpected statistics error and achieve a more convincing result. It generally took us less than 1 h in computation time to complete the



Fig. 10. Sensitivity coefficients of PCoT (a) and ranks of PCoT (b).

1000 sample calculations for each case.

In the current work, the uncertainty analysis was performed for both steady-state conditions and transient scenarios by generating 1000 sample inputs through randomly perturbing all input parameters of interest simultaneously according to their prescribed uncertainty information. The sample calculation would then be executed by feeding each sample input to RELAP5-3D either in series or in parallel depending on the paradigm. The output data for each sampled calculation was dumped to a CSV formatted file, which contains the FOM responses predefined by the user. A MATLAB based processing script was used to statically analyze these output data stored in 1000 CSV files and show the results graphically. In the BEPU framework developed in this work, the uncertainty analysis procedure is automated in the RAVEN platform and the data flow involved in the procedure is clearly explained in Fig. 3.

4.2.1. Steady-state conditions

Fig. 11 shows the probability distributions of PCT and PCoT, obtained from the uncertainty analysis on the steady-state conditions. These results were extracted from 1000 RELAP5-3D outputs and depicted as histogram plots. The corresponding Gaussian distribution curves fitted through the histogram data are highlighted with solid curves in Fig. 11.

Table 6 summarizes the standard statistics characteristics of the PCT and PCoT distributions. It can be seen that although there exists a small discrepancy between the results of RELAP5-3D and PARET (See Table 3), the PCT and PCoT predictions from PARET are all within the 95/95 confidence level (C.L.). For reactor safety, the PCT must not reach the fuel blister temperature, which is 515 °C–575 °C for uranium silicide LEU fuel (NRC, 1988). As shown in Table 6, the maximum peak cladding temperature observed in the analysis (120.45 °C) is way lower than the limiting value, which indicates the design has a considerably large safety margin at the steady-state operation conditions.

4.2.2. Reactivity insertion accident

The uncertainty analysis procedure for the RIA case is similar to the steady-state case except transient calculations were performed in this case. A set of 1000 sample inputs were generated by randomly perturbing two input parameters (inlet coolant temperature and mass flow rate) simultaneously according their prescribed uncertainty range given Table 6

Statistics characteristics of PCT and PCoT.

	PCT [°C]	PCoT [°C]
Mean	90.10	54.25
Standard Dev.	9.75	13.89
95% Lower C.L.	89.49	53.38
95% Upper C.L.	90.71	55.12
Maximum	120.45	95.25

in Table 2. Note only two input parameters were considered in the RIA case because the reactor power became an important output variable in this case. The RELAP5-3D was employed through the RAVEN platform to execute all sample calculations. After the calculations, the output files were then post-processed and the FOM responses were analyzed to reveal the uncertainty characteristics.

Fig. 12 shows the baseline results and uncertainties of the peak cladding and peak coolant temperature in the hot channel evolved in the RIA through the uncertainty analysis. The bassline results are basically copied from the conventional safety analysis (See Fig. 6), and the uncertainties are illustrated with a grey strap surrounding the baseline plots. As shown in Fig. 12, the maximum cladding temperature does not exceed the fuel blister temperature (515 °C–575 °C for the uranium silicide LEU fuel) all through the RIA period. The results also indicate the PCT and PCoT predicted by PARET are mostly within the range of the uncertainties. Moreover, the widths of uncertainty straps for the PCT and PCoT are appeared to be stabilized at 7.5 °C after the temperatures become steady into the RIA.

4.2.3. Loss of flow accident

The uncertainty analysis results of the LOFA case were achieved in the same way as that of the RIA, except this time the reactor power was also considered as an input parameter and simultaneously perturbed with prescribed $\pm 2\%$ uncertainty range to generate the sample input. Fig. 13 illustrates the baseline results and uncertainties of peak cladding and peak coolant temperature evolved in the LOFA. The uncertainties are illustrated with a grey strap surrounding the baseline plots, which are copied from the conventional safety analysis (See Fig. 8). As shown in Fig. 13, the PCT and PCoT predicted by PARET mostly stay within the



Fig. 11. Histogram views of PCT (a) and PCoT (b) in the steady-state.



Fig. 12. Baseline results and uncertainties of PCT (a) and PCoT (b) in the RIA.



Fig. 13. Baseline results and uncertainties of PCT (a) and PCoT (b) in the LOFA.

range of the uncertainties before the establishment of stabilized natural circulation, which occurred at ~ 5 s into the transient. However, the discrepancy of the temperature predictions by two codes after the flow reversal point (~ 5 s into the transient) cannot be well interpreted by the uncertainties determined in this study. The temperatures by PARET decreased sharply after the flow reversal, while temperatures by RELAP5-3D decreased rather smoothly. This difference could be possibly attributed to some epistemic uncertainty sources such as the difference of natural circulation models in both codes.

4.3. Remarks on the BEPU analysis

As demonstrated by the case study problem, compared to the conventional transient analysis discussed in Section 3, a lot of additional useful information can be obtained through the BEPU analysis. Specifically, sensitivity analysis informs the insights and influencing level of each individual input parameters on the FOM of interest by evaluating and comparing sensitivity coefficients of the FOM responses with respect to input parameters. For example, through the sensitivity analysis at the steady state conditions, the sensitivity of PCT to the mass flow rate and reactor power have shown a nearly linear variation characteristic, while the sensitivity to the inlet temperature has exhibited some non-linearity features. Furthermore, it was also found that the inlet coolant temperature is the most influencing factor to PCT compared other two parameters. One the other hand, uncertainty analysis provides a better interpretation to predictions of physics model and discrepancies between two different models. For example, the uncertainty analysis at the steady-state conditions reveals that although there exists a small discrepancy between RELAP5-3D and PARET, but the PCT and PCoT predictions from PARET are all within the 95/95 confidence level, and the maximum value of peak cladding temperature does not exceed the limiting value of the safety restriction. The uncertainty analysis for LOFA shows the PCT predicted by PARET is mostly within the uncertainty range of RELAP5-3D predictions before the establishment of natural circulation but deviates largely after the flow reversal, which may lead us to examine other uncertainty sources such as the difference of natural circulation models in both codes.

5. Conclusions

This paper presents a BEPU safety analysis framework for research reactors by using physics models in RELAP5-3D and sensitivity uncertainty analysis capabilities of RAVEN. The main purpose of the framework development is to integrate the response uncertainties into the physics predictions during the course of safety transient analysis such that reactor design decisions can be made in a more risk-informed environment. The conceptual NIST research reactor design was employed as a case study example in the paper to facilitate the demonstration of the BEPU framework development.

In order to illustrate the possible predictive deficits that existed in the conventional transient analysis method, the case study example was modeled and studied by two different T/H transient analysis codes, namely the RELAP5-3D and PARET, following a standard conventional transient analysis procedure. Steady-state conditions as well as two typical design-basis accidental scenarios, RIA and LOFA, were examined in the analyses to identify and highlight the potential drawbacks that existed in the conventional safety analysis practices. These shortages provide a driving source for us to proceed with the BEPU analysis using the BEPU framework developed in the current study.

There were two additional tasks performed in the BEPU, namely the sensitivity analysis and uncertainty analysis. Through the sensitivity analysis, we obtained the sensitivity coefficients of the FOMs with respect to the parameters of interest to see insights on the levels of influence to the FOMs by different parameter. In the case study, sensitivity analysis performed at the steady-state conditions revealed that sensitivity coefficients of PCT and PCoT to the mass flow rate and reactor power showed a nearly linear variation characteristic, while the sensitivity to the inlet temperature exhibited some non-linear feature. The dimensionless sensitivity coefficients of PCT to inlet coolant temperature, reactor power, and inlet coolant mass flow rate were 0.572, 0.133, and -0.112, respectively. The sensitivity coefficients for PCoT to each parameter were 0.857, 0.03 and -0.03, respectively. This indicated that the inlet coolant temperature was the dominant factor that affects PCT and PCoT and the impact of the reactor power and inlet coolant mass flow rate was relatively small.

Through the uncertainty analysis under the BEPU framework, we determined the uncertainty in output results by using a better interpretation and implementation of the variability contributed by model parameter uncertainty. In the case study, uncertainty analysis was performed on two common design-basis accidents, RIA and LOFA, and revealed that the PCT and PCoT predicted by PARET were mostly within the range of the prediction uncertainties by RELAP-5D during the RIA. Similar results were observed for the LOFA before the period of the stabilized natural circulation, but an outlier discrepancy after the flow reversal point was noticed and could not be explained by the current uncertainty analysis (which focus on aleatoric uncertainties). This may be caused by other epistemic uncertainty resources such as the differences of natural circulation models in both codes. The uncertainty analysis also provided additional uncertainty information that confirmed sufficient safety margins for the case study reactor design during both RIA and LOFA situations, which further justified the advantages and benefits of the BEPU framework developed in this work.

Credit author statement

Tao Liu.: Investigation, Resources, Data curation, Formal analysis, Writing – original draft preparation, **Zeyun Wu**: Supervision, Project administration, Conceptualization, Methodology, Writing- Reviewing and Editing, **Cihang Lu**: Methodology, Writing- Reviewing and Editing, **Robert Martin**, Supervision, Writing- Reviewing and Editing

Declaration of competing interest

The authors declare that they have no known competing financial

interests or personal relationships that could have appeared to influence the work reported in this paper.

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