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# Validation of the 1-D Thermal Stratification Model in Gallium Environment

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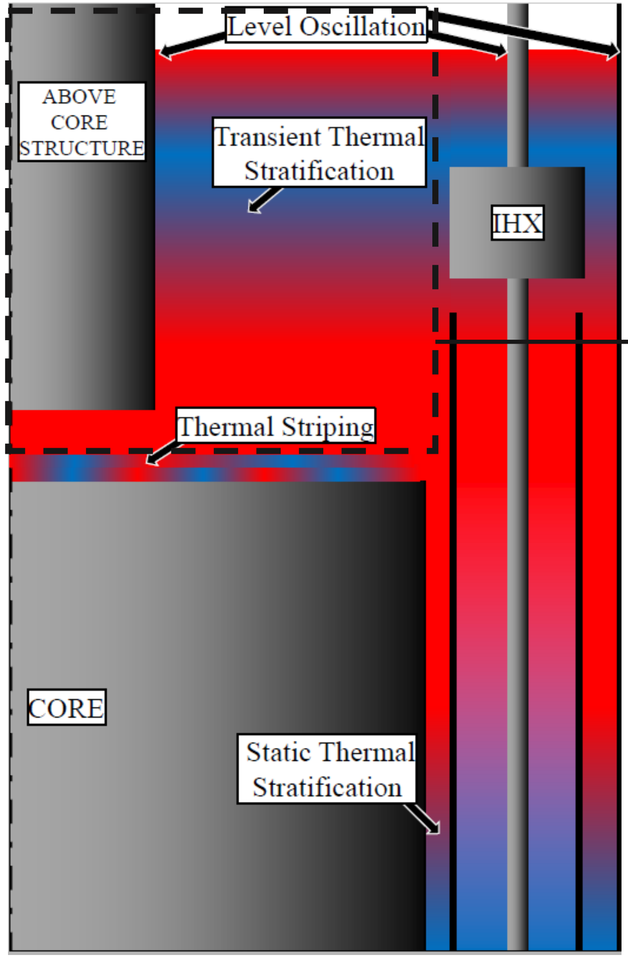
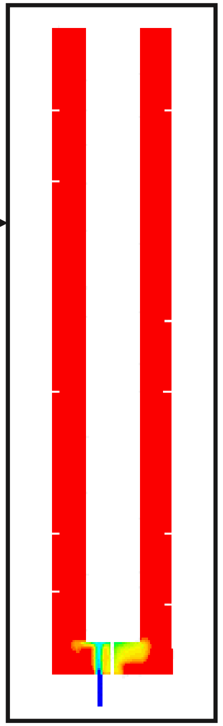
# Objective

**To validate the improved 1-D thermal stratification model against experimental data acquired by using Gallium as the working fluid.**

# Thermal stratification in nuclear systems

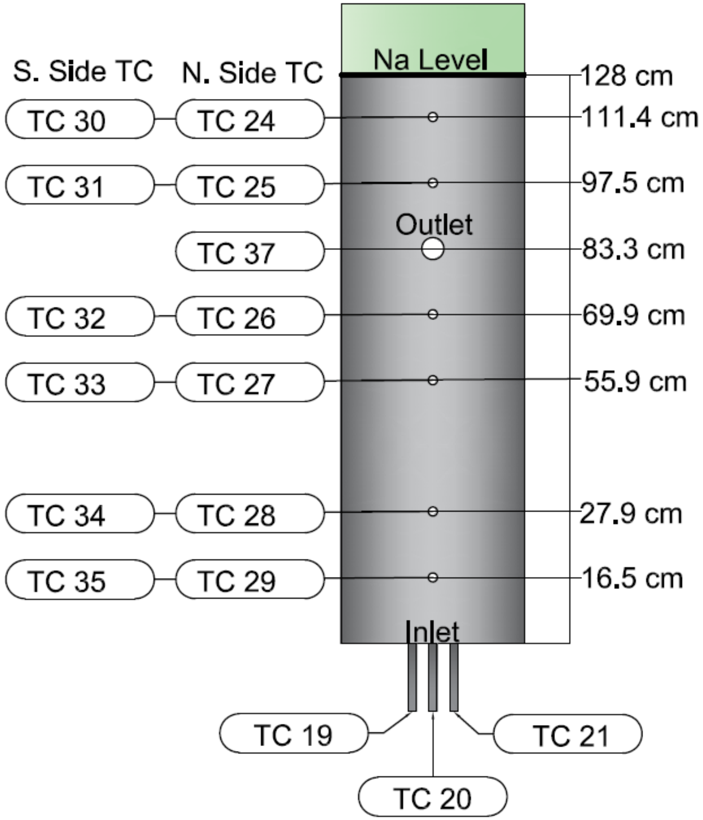
- Thermal stratification
  - Formation of stratified layers of coolant with a large temperature gradient
- Nuclear systems involved
  - High-Temperature Gas-Cooled Reactors (HTGR)
  - Small-Modular Boiling-Water Reactors (SMBWR)
  - **Pool-type Sodium-Cooled Fast Reactors (SFR)**
  - ...
- Concerns
  - Leads to neutronic and thermal-hydraulic instabilities
  - Causes thermal fatigue crack growth
  - **Impedes natural circulation**

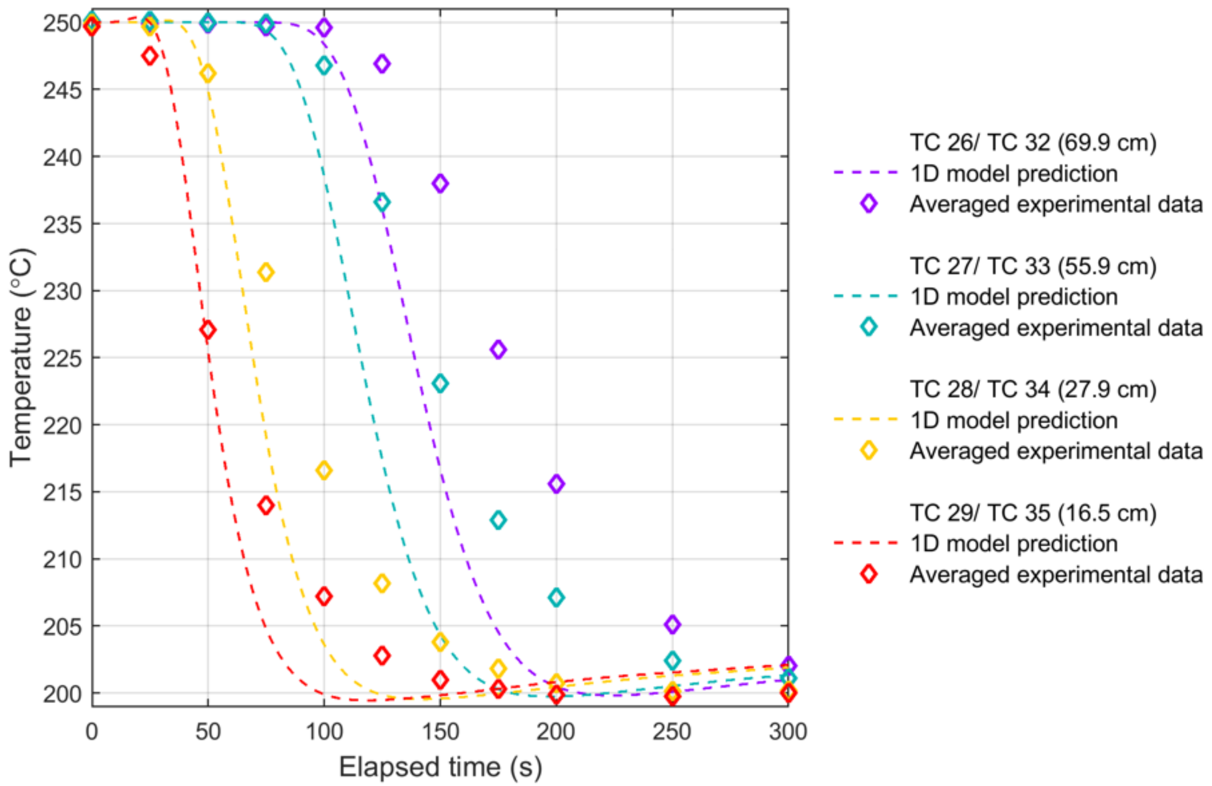


# Preliminary 1-D thermal stratification model (1)

## The Thermal Stratification Testing Facility (TSTF)




- University of Wisconsin-Madison
- Cylindrical with H=150 cm and D=32 cm
- Na as working fluid

- Default parameters
- Tested by 9 data sets from TSTF
- Reasonable performance

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# Preliminary 1-D thermal stratification model (2)

(Peterson, 1994)

$$A_{amb}(z)\frac{\partial \rho_{amb}}{\partial t} + \frac{\partial(\rho_{amb}Q_{amb})}{\partial z} = \sum_{k=1}^{N_{jet}} \rho_k Q'_k \ (conservation\ of\ mass)$$

$$\frac{\partial P_{amb}}{\partial z} = -\rho_{amb} g \ (conservation\ of\ momentum)$$

$$A_{amb}(z)\frac{\partial(\rho_{amb}h_{amb})}{\partial t} + \frac{\partial(\rho_{amb}h_{amb}Q_{amb})}{\partial z} - A_{amb}(z)\frac{\partial}{\partial z}\left(k_{amb}\frac{\partial T_{amb}}{\partial z}\right) = \sum_{k=1}^{N_{jet}} \rho_k h_k Q'_k \ (conservation\ of\ energy)$$

By combining the mass and the energy equations

$$\rho_{amb}c_p\frac{\partial T_{amb}}{\partial t} + \rho_{sf}c_p\bar{u}_z\frac{\partial T_{amb}}{\partial z} - \frac{\partial}{\partial z}\left(k_{amb}\frac{\partial T_{amb}}{\partial z}\right) = \frac{1}{A_{amb}(z)}\sum_{k=1}^{N_{jet}}(\rho Q')_k\,(h_k - h_{amb})$$

By approximating $dh = c_p dT$ and $\Delta h = c_p \Delta T$ when $T_{jet} \approx T_{amb}$

$$\rho_{amb}c_p\frac{\partial T_{amb}}{\partial t} + \rho_{sf}c_p\bar{u}_z\frac{\partial T_{amb}}{\partial z} - \frac{\partial}{\partial z}\left(\boxed{k_{amb}}\frac{\partial T_{amb}}{\partial z}\right) = \frac{N_{jet}}{A_{amb}}c_{p,jet}\rho_{jet}\boxed{Q'_{jet}}(T_{jet} - T_{amb})$$

Effective thermal conductivity

Linear jet dispersion rate

# Parameters to be calibrated

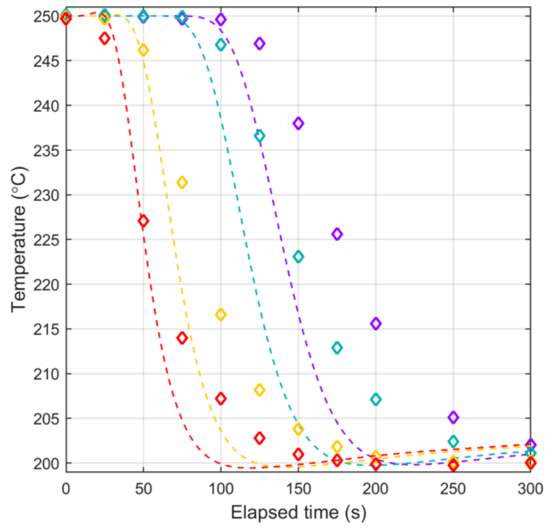
$$\rho_{amb} c_p \frac{\partial T_{amb}}{\partial t} + \rho_{sf} c_p \bar{u}_z \frac{\partial T_{amb}}{\partial z} - \frac{\partial}{\partial z}\left(\boxed{k_{amb}} \frac{\partial T_{amb}}{\partial z}\right) = \frac{N_{jet}}{A_{amb}} c_{p,jet} \rho_{jet} Q'_{jet} \left(T_{jet} - T_{amb}\right)$$

Effective thermal conductivity (turbulence-enhanced )

where

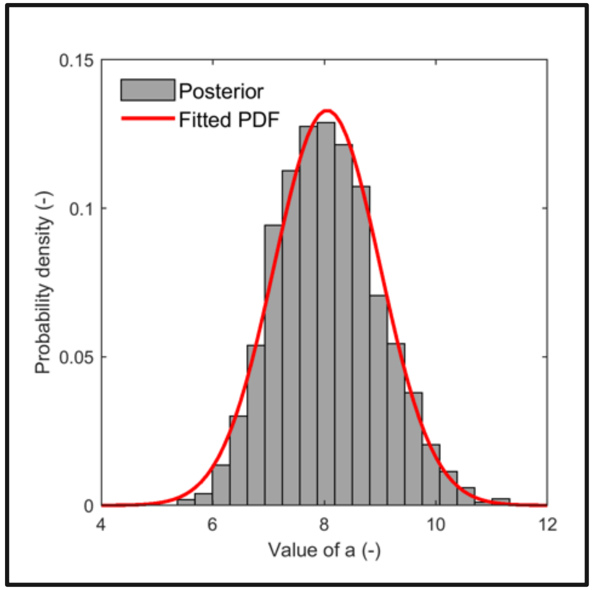
- Effective thermal conductivity $k_{amb} = a\left(\frac{Re_\tau}{Ri}\right)^b \cdot k_c$ (Shih et al., 2005)
- Turbulent Reynolds number $Re_\tau = \frac{\rho k^2}{\mu \epsilon}$ (Jones and Launder, 1973)
  - $k = 0.01 U_{jet}^2$ (Lai et al., 1986)
  - $\epsilon = \frac{2k^{3/2}}{d_{jet}}$ (Lai et al., 1986)
- Richardson number $Ri_{amb} = \frac{g}{\rho_{amb}} \frac{\partial \rho_{amb}/\partial z}{(\partial u_z/\partial z)^2}$
- Parameters to be calibrated:
  - $a$
  - $b$

# Development of the 1-D thermal stratification model




Preliminary 1-D TS model
- Default parameters
- Tested by 9 data sets from TSTF
- Reasonable performance

(Lu et al., 2020)




Improved 1-D TS model
- Inverse Uncertainty Quantification (UQ)
- Trained by 4 data sets from TSTF

# Bayesian based inverse UQ method

$$\underbrace{p(\boldsymbol{\theta}^*|\boldsymbol{y}^E,\boldsymbol{y}^M)}_{\text{Posterior}} \propto \underbrace{p(\boldsymbol{\theta}^*)}_{\text{Prior}} \cdot \underbrace{\frac{1}{\sqrt{|\boldsymbol{\Sigma}|}}\exp\left[-\frac{1}{2}[\boldsymbol{y}^E-\boldsymbol{y}^M-\delta]^T\boldsymbol{\Sigma}^{-1}[\boldsymbol{y}^E-\boldsymbol{y}^M-\delta]\right]}_{\text{Likelihood function}}$$

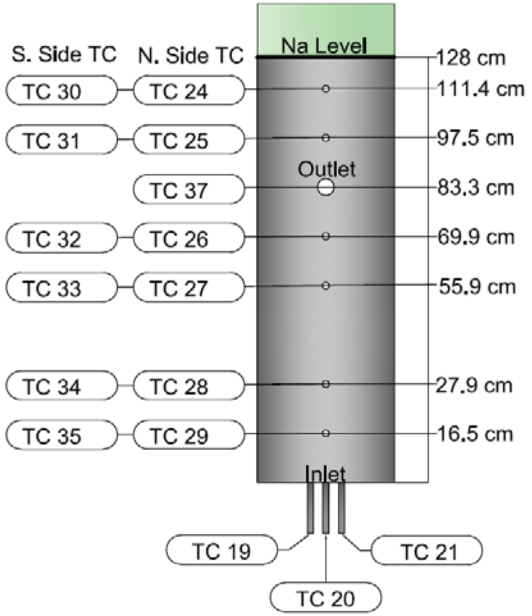
- Prior PDF: The degree of the belief of the values of the true values $\boldsymbol{\theta}^*$ for the calibration parameters, before observing the experimental data $\boldsymbol{y}^E$.
- Posterior PDF: The degree of the belief of the values of the true values $\boldsymbol{\theta}^*$ for the calibration parameters, after observing the experimental data $\boldsymbol{y}^E$.
- Likelihood: Determined by the model, experimental settings, experimental data, etc.
- Posterior PDF explored by the Markov Chain Monte Carlo (MCMC) sampling method.
- Tool: **QULab** (Marelli and Sudret, 2014).

# Parameter calibration

Prior
$a \sim U[0, 50]$
$b \sim U[0, 1]$

+

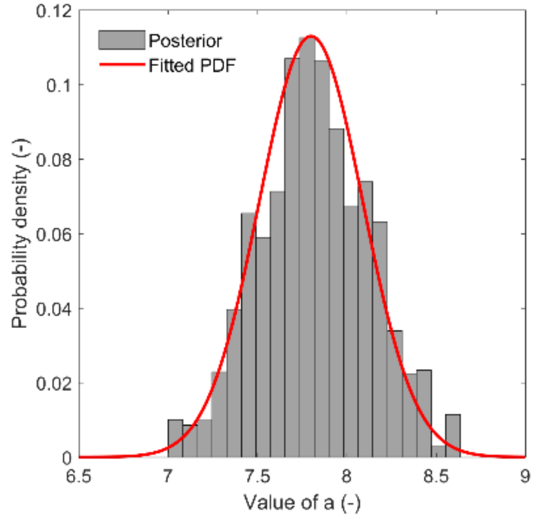
Likelihood function



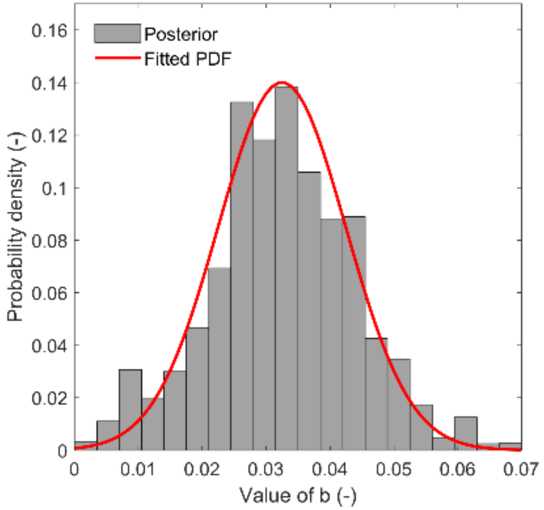

4 data sets from TSTF

$4 \cdot 10^5$ samples $\Rightarrow$

Posterior




$a \sim N(\mathbf{7.8}, 0.085)$




$b \sim N(\mathbf{0.032}, 10^{-4})$

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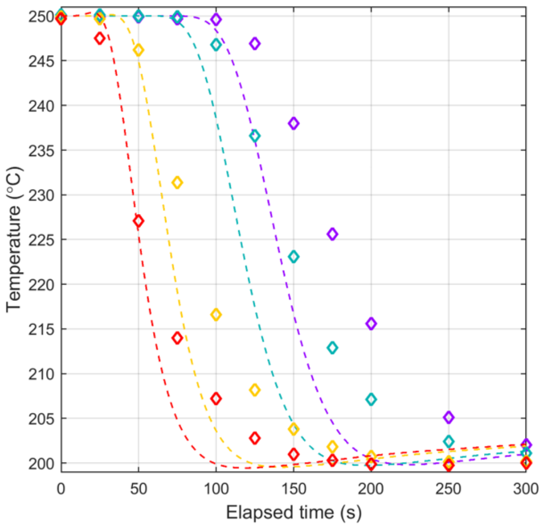
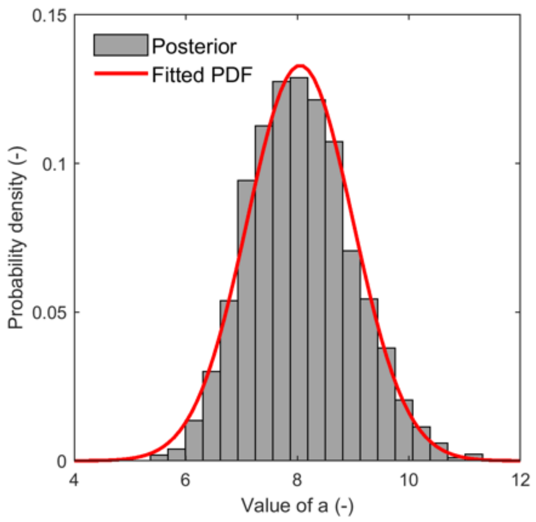
# Development of the 1-D thermal stratification model




Preliminary 1-D TS model

- Default parameters
- Tested by 9 data sets from TSTF
- Reasonable performance

(Lu et al., 2020)

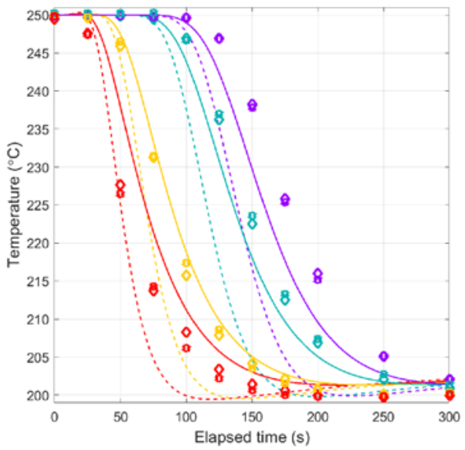



Improved 1-D TS model

- Inverse Uncertainty Quantification
- Trained by 4 data sets from TSTF

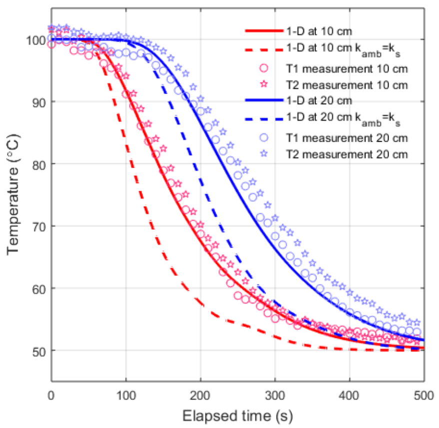
Validation

(Lu et al., submitted)




5 data sets from TSTF

This presentation

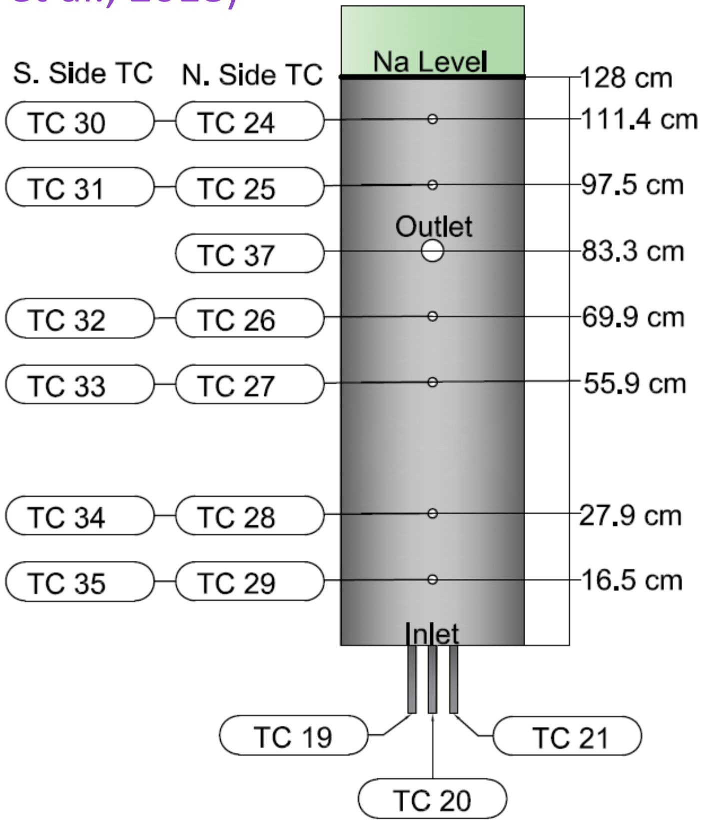



5 data sets from GaTE

# Experimental apparatus

## The Thermal Stratification Testing Facility

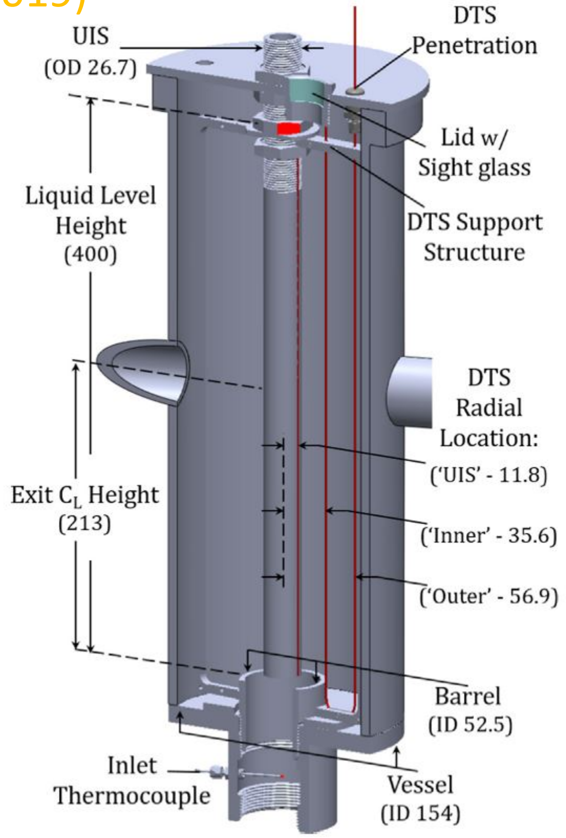
(Schneider et al., 2018)




- ➢ University of Wisconsin-Madison
- ➢ Cylindrical with H=150 cm and D=32 cm
- ➢ Na as working fluid

## The Gallium Thermal-hydraulic Experiment

(Ward et al., 2019)




- ➢ Kansas State University
- ➢ Cylindrical with H=40 cm and D=15 cm
- ➢ Ga as working fluid

# T-H properties

*Melting point*:

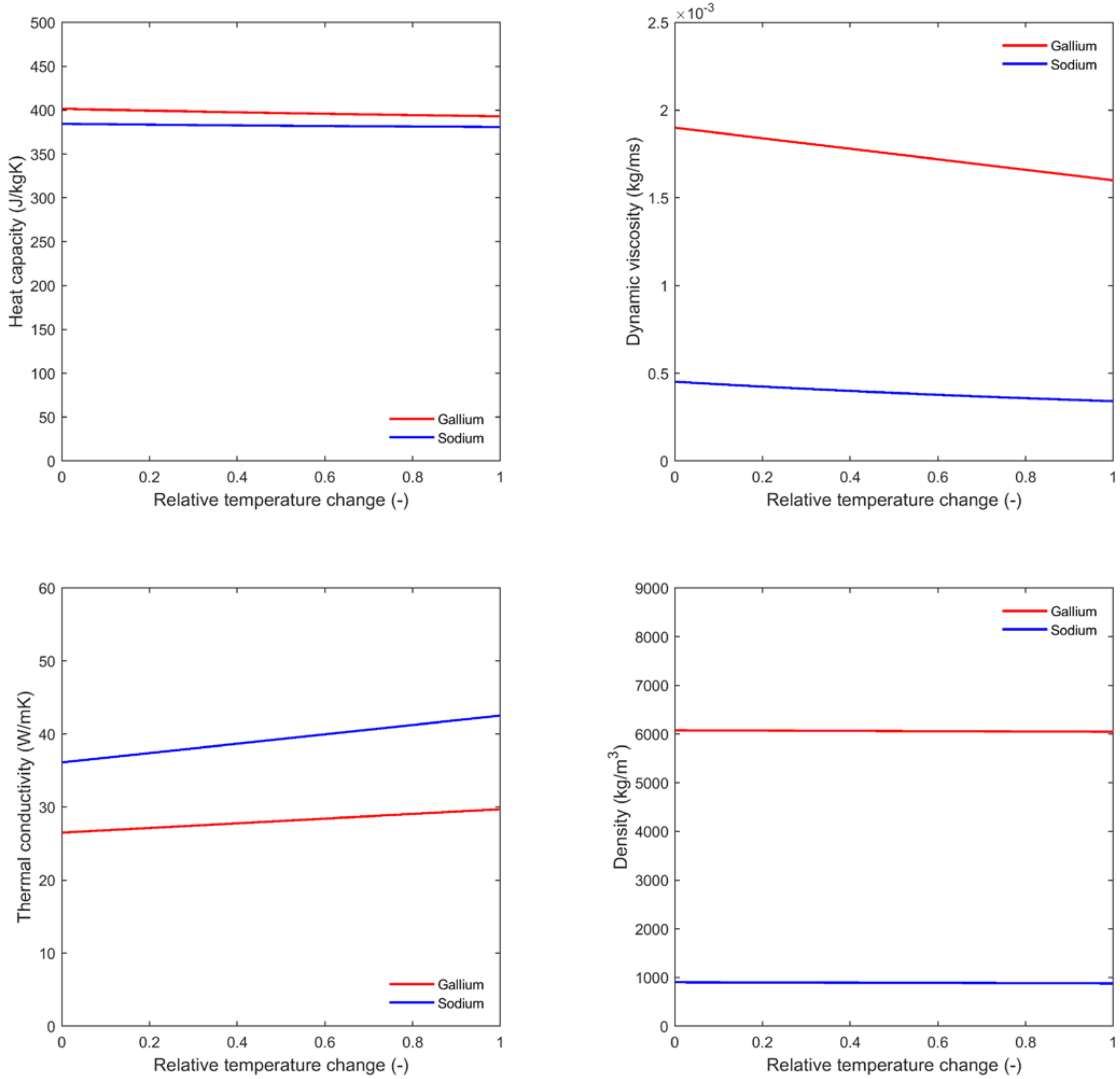
- $Na \sim 100°C$
- $Ga \sim 30°C$

*Experimental temperature range*:

- $Na\ 200°C \sim 300°C$
- $Ga\ 50°C \sim 100°C$

*Properties within temperature ranges*:

- $C_p: Ga \sim Na$
- $k : Ga \sim Na$
- $\mu : Ga \sim 4Na$
- $\rho : Ga \sim 6Na$

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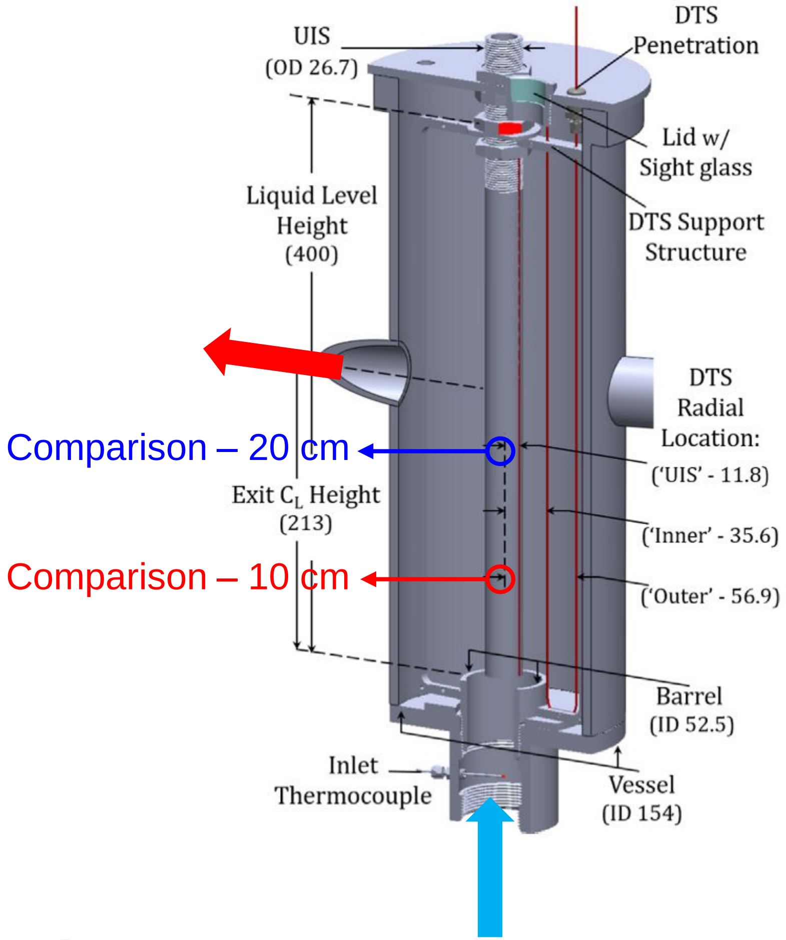
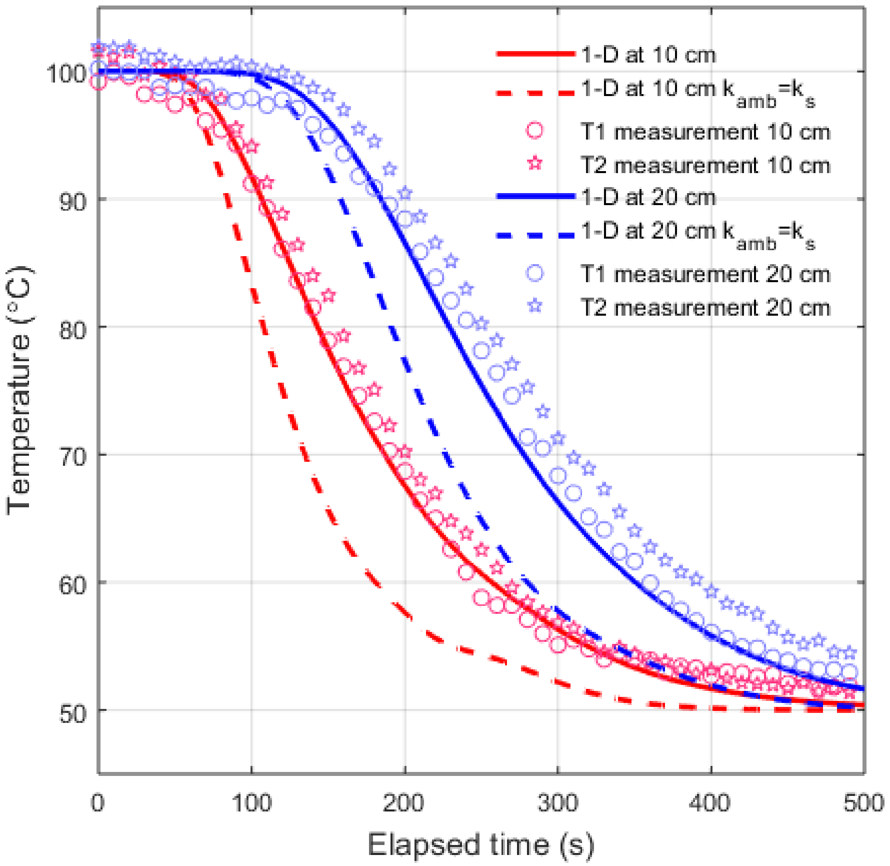
# Performance of the improved 1-D T-S model (1)



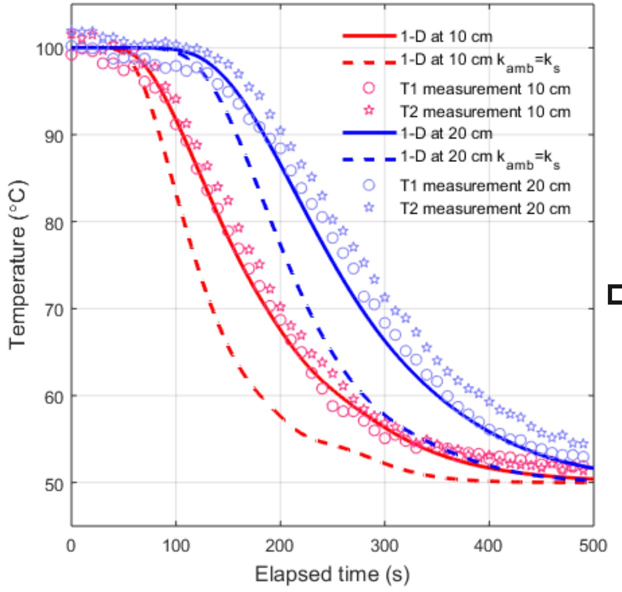

Initial temperature - 100 °C
Jet temperature - 50 °C

| Test # | jet velocity (mm/s) |
|---|---|
| 1 | 10 |
| 2 | 20 |
| 3 | 40 |
| 4 | 60 |
| 5 | 80 |



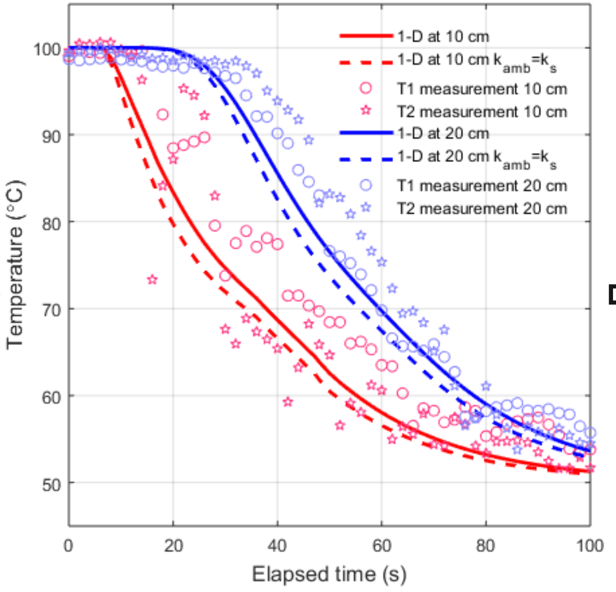

Test #1

# Performance of the improved 1-D T-S model (2)



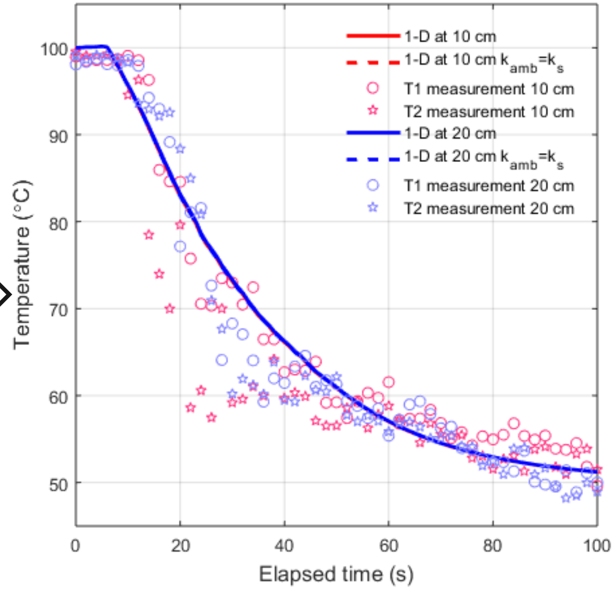

Test #1

$Q_{jet} \uparrow$




Test #3

$Q_{jet} \uparrow$




Test #5

| Test # | Jet velocity (mm/s) | Maximum Error (°C) | | | Averaged Error (°C) | | |
|---|---|---|---|---|---|---|---|
| | | Before | After | Change | Before | After | Change |
| 1 | 10 | 17 | 5 | -71% | 6 | 1.5 | -75% |
| 2 | 20 | 14 | 3.5 | -75% | 6.5 | 1.5 | -77% |
| 3 | 40 | 18.5 | 15 | -19% | 4.5 | 3.5 | -22% |
| 4 | 60 | 16.5 | 16.5 | 0% | 4 | 4 | 0% |
| 5 | 80 | 24.5 | 24.5 | 0% | 5.5 | 5.5 | 0% |

# Summary

- The correlation of $k_{amb}$, trained with the sodium data, had reasonable performance in the gallium environment thanks to the non-dimensional numbers used in terms of its establishment.
- This demonstrated the capability of the improved 1-D model to work for different facilities using different working fluid.

# Future work

- In this work, the $Re_{\tau}/Ri$ was only calculated at the beginning of the problem and assumed to be identical at all axial levels. For a better performance, in future work we will calculate these non-dimensional numbers at each node, and have them updated throughout the calculation.
- To improve the jet model.

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